Reinstatement Provisions in Catastrophe Reinsurance: The Contract Economics of Post-Loss Capacity*

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Abstract. Reinstatement provisions restore the limit of a catastrophe excess-of-loss treaty after a loss, on terms fixed at inception. The actuarial literature prices the clause. This paper asks why a cedent would want it. In a two-period model the cost of reinsurance capital after a first event is random, a spot market reprices cover at that cost, and the cedent chooses how much restoration to commit in advance and how much to buy afterwards. A restoration paid for on delivery at the expected spot price is a contingent forward on post-loss capacity. It has value if and only if the post-loss price is uncertain. The cedent commits more than it would buy in the dearest market and less than in the cheapest, and never the full limit. Prepayment adds a premium-smoothing motive and raises the committed amount. Full, binding restoration does not always dominate spot repurchase, and optimal restoration is smaller on remote layers. Pre-funded, collateralised restoration is dearer except on high-frequency layers. The model predicts prepaid restoration and does not explain the customary reinstatement premium.

JEL classification: G22, D86, D81, G32. **Keywords:** reinstatement provisions; excess-of-loss reinsurance; contract design; catastrophe risk; post-loss capacity; forward contracts.

1. Introduction

Most catastrophe excess-of-loss treaties contain a provision that the theory of insurance contracts has not examined. When a loss consumes the limit, the cover does not end. The limit is restored, or reinstated, for the rest of the contract period against an additional premium fixed at inception. The premium is customarily quoted as a percentage of the original premium and is payable in proportion to the limit consumed (Clark 1996; Albrecher, Beirlant, and Teugels 2017). A substantial actuarial literature prices the clause, beginning with Simon (1972) and Sundt (1991). That literature takes the contract form as given. Why a cedent would want capacity to be restored on pre-agreed terms, and how much of it, is a question about contract design that has not been asked.

A natural answer starts from the market the cedent would otherwise face. After a large event, reinsurance prices rise above their pre-event level and fall back only as capital re-enters (Winter 1994; Gron 1994; Froot and O'Connell 1999; Froot 2001). A cedent whose limit has just been consumed must either go without cover for the rest of the period or buy it in that market. The reinstatement fixes the terms of the second purchase before the first event occurs. In this sense

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it resembles guaranteed renewability in health and life insurance, where a premium commitment protects the insured against repricing (Pauly, Kunreuther, and Hirth 1995; Cochrane 1995; Hendel and Lizzeri 2003). The difference is the source of the repricing: the state of the market's capital, not information about the insured.

Stating the idea is easier than making it work in a model. If the post-event price is higher but known in advance, a competitive reinsurer that commits to restore the limit must charge for the capacity it will have to supply at that price, and the commitment gives the cedent nothing it could not obtain by buying later. A cedent that buys later can also choose how much to buy, and at a loaded price it will not buy the full limit (Mossin 1968). A binding restoration takes that choice away. An account of the clause has to say what the commitment adds and what the loss of flexibility costs.

This paper sets out a model in which both can be measured. A contract period has two sub-periods and at most one event in each. Reinsurers are competitive and risk neutral, and every limit must be backed by capital. The single friction is that the cost of capital after a first event is random: it is drawn from a known distribution once the event has occurred. Cover for the second sub-period then trades in a spot market at the realised cost. At inception the cedent can commit to a restoration of any size up to the limit, pay for it in advance or on delivery, and top it up at spot after the event. The contract cannot depend on the realised cost of capital. A single contract family therefore contains the bare limit with spot repurchase, the standard reinstatement clause, and everything between them.

Three results form the core. First, a restoration paid for on delivery at the expected spot price is a contingent forward on post-loss capacity, and it has value if and only if the post-loss price is uncertain (Proposition 1). With a known price it replicates spot repurchase. With an uncertain price, a cedent that would buy at spot in every state commits strictly more than it would buy in the dearest market and no more than it would buy in the cheapest. It tops up when capacity turns out cheap and holds more cover than it would have bought when capacity turns out dear. The committed amount is always below the limit. Second, paying for the restoration in advance adds a separate motive, the smoothing of premium across states, which operates even when the post-loss price is known and which raises the committed amount (Proposition 2). The cedent's preferred reinstatement premium is zero. Third, the standard clause, which restores the full limit and binds both parties, is not always preferred to spot repurchase (Proposition 3). The comparison is given in closed form. A rise in the cost of post-event capital favours the clause only while the cedent would still buy a sufficient amount at spot. Once capacity is dear enough that it would buy little, further scarcity favours the spot market.

Three implications follow. Committed restoration falls when post-event capital becomes dearer across the board, and the bounds on it fall as the layer becomes more remote, which reproduces the pattern of fewer reinstatements on higher layers without assuming it (Proposition 4). A fully collateralised seller can offer restoration only by holding the second limit from inception, and this is dearer than contingent balance-sheet capacity except on high-frequency layers or under extreme scarcity (Proposition 5). A restoration that the cedent may decline is taken up in every state only if the reinstatement premium is below an explicit bound. A cedent that bears retained losses after an event commits more, and restores the full limit if those losses

are large enough (Proposition 6).

The model also has a negative result that I state plainly. A cedent with a concave value function prefers to prepay for restoration in full. The customary reinstatement premium of 100 per cent of the original premium is therefore not explained by risk sharing under costly capital. Section 7 discusses the frictions that might explain it and what the model implies about where a given rate is most costly.

The contribution is to treat the reinstatement as an object of contract theory. The paper connects three literatures: optimal insurance design (Arrow 1974; Raviv 1979), the capacity-constraint account of reinsurance pricing (Winter 1994; Gron 1994; Froot 2001), and the theory of forward contracting under price uncertainty (Holthausen 1979; Feder, Just, and Schmitz 1980). Section 2 describes the provision. Section 3 reviews the literature. Section 4 sets out the model. Section 5 states the results, and Section 6 illustrates them numerically. Section 7 discusses the institutional record and the limits of the analysis. Section 8 concludes. All proofs are in the Appendix.

2. Reinstatements in practice

A catastrophe excess-of-loss treaty indemnifies the cedent for the part of an event loss that exceeds an attachment point, up to a limit per occurrence. When an event consumes all or part of the limit, a reinstatement provision restores the consumed amount for the remainder of the treaty period. The cedent pays a reinstatement premium that is fixed in the treaty at inception. It is usually quoted as a percentage of the original premium, so that "one reinstatement at 100 per cent" means one restoration at the full original premium, and it is payable pro rata as to amount, that is, in proportion to the share of the limit reinstated. Some contracts also prorate for the unexpired time. The premium is commonly settled by offset against the loss recovery, so that the provision operates without negotiation after the loss (Clark 1996; Albrecher, Beirlant, and Teugels 2017).

Three features of practice are relevant to the theory. The first is that restoration is automatic and binding. Neither party decides after the event whether the limit is restored. The second is a cross-section over layers. Low layers with frequent losses typically carry several reinstatements, sometimes at reduced or no additional premium. Catastrophe layers typically carry one or two at 100 per cent. The highest layers of large programmes often carry one or none (Clark 1996; Albrecher, Beirlant, and Teugels 2017). The third concerns collateralised capacity. Catastrophe bonds and collateralised reinsurance are generally written without reinstatement provisions (Cummins 2008; Artemis 2017). A collateralised seller that offered a restoration would have to fund the restored limit in the trust like the first.

The actuarial literature answers the question "at what rate" for a given clause. An economic account should say why the commitment is demanded, how much restoration a cedent would choose, how the choice varies across layers, and why sellers with different capital structures differ in their ability to supply it.

3. Related literature

The pricing of excess-of-loss reinsurance with reinstatements has been studied since Simon (1972). Sundt (1991) derives pure premiums and premiums under the standard deviation principle for layers with free or paid reinstatements. Walhin and Paris (2000, 2001) compute the adjustment coefficient and the ruin probability of the ceding company for such treaties with a multivariate recursion. Mata (2000) obtains the joint distribution of losses for consecutive layers and compares premium principles. Hess and Schmidt (2004) characterise the premium plan that minimises the expected squared difference between the reinsurer's loss and its premium income. Hürlimann (2005) gives distribution-free approximations to pure premiums. Anderson and Dong (1998) price reinstatement provisions with a catastrophe model, and Campana (2006) studies reinstatement premiums. Albrecher and Haas (2011) study the finite-time ruin probability of an insurer that buys such treaties. Kazi-Tani (2021) is the closest to the present paper in spirit: it prices a treaty with reinstatements by indifference, using a monetary utility function and a risk measure to capture the risk reduction and the capital relief that reinsurance provides. In all of these contributions the number and price of reinstatements are parameters of a given contract. None asks which contract a cedent would choose when the alternative is to buy cover after the loss.

The literature on optimal insurance design characterises the indemnity schedule for a single loss and a single contracting date (Borch 1962; Arrow 1974; Raviv 1979; Huberman, Mayers, and Smith 1983). With one event there is nothing to restore. The model below is a minimal extension to two events in which the price of cover for the second differs from the price for the first. One classical result carries over and does much of the work: at a price above the actuarially fair one, cover is partial (Mossin 1968).

The price path that motivates the clause comes from the capacity-constraint literature. Winter (1994) and Gron (1994) show that insurance prices rise when industry capital is depleted, because external capital is costly to raise. Froot and O'Connell (1999) and Froot (2001) document the rise in catastrophe reinsurance prices after large events, and Zanjani (2002) shows how the cost of capital allocated to tail states enters catastrophe pricing. I take from this literature a single reduced-form object, the distribution of the cost of capital after an event. The demand side follows Froot, Scharfstein, and Stein (1993): an insurer for which external finance is costly values internal funds by a concave function.

Forward contracting under price uncertainty has been analysed for a producer that sells output at a random price (Holthausen 1979; Feder, Just, and Schmitz 1980). The reinstatement differs from the forward in those models in three ways. The cedent is a buyer. The contract is conditional on an event. The committed quantity cannot be resold, so an over-committed cedent holds cover it would not have bought. The last feature is the source of the upper bound on committed restoration in Proposition 1.

The renewability literature shows that premium commitments insure against reclassification (Pauly, Kunreuther, and Hirth 1995; Cochrane 1995; Hendel and Lizzeri 2003). The analogy with the reinstatement is useful but partial. In those models the insured's own type changes. Here the price of capacity changes for every buyer, and the commitment is valuable only to the extent that this price is uncertain when the contract is written.

Finally, Lakdawalla and Zanjani (2012) analyse the trade-off between collateralised and balance-sheet risk transfer: collateral removes default risk but ties capital to a single exposure. Proposition 5 applies the same trade-off to the second limit of a treaty. Cummins (2008) and Cummins and Weiss (2009) survey the securitised market.

4. The model

4.1 Environment

A contract period consists of two sub-periods, $t = 1, 2$. In sub-period t an event occurs with probability $p_t \in (0, 1)$, independently across sub-periods. An event causes a loss $L > 0$ to the layer under consideration. Losses to the cedent outside the layer are introduced in Section 5.6 and are otherwise set to zero.

The cedent has initial funds W_0 and values terminal funds w according to

$$V(w) = -e^{-aw}, \qquad a > 0.$$

Concavity stands for costly external finance: an insurer whose internal funds are depleted must raise capital on unfavourable terms or forgo business, so the marginal value of internal funds is decreasing (Froot, Scharfstein, and Stein 1993). The exponential form removes wealth effects from the decision the cedent takes after an event and yields closed forms. Section 7 says which results depend on it.

Reinsurers are competitive and risk neutral, and contracts are enforceable. In each sub-period a reinsurer must hold capital equal to the largest indemnity that can fall due under its contract in that sub-period. Capital has a cost per unit and per sub-period. In sub-period 1, and in sub-period 2 if no event has occurred in sub-period 1, the cost is $\kappa > 0$. If an event has occurred in sub-period 1, the cost of capital in sub-period 2 is a random variable k with distribution function F on $[k_\ell, k_h]$, where $0 < k_\ell \leq k_h$ and $p_2 + k_h < 1$, and with mean $\bar{k}$. The realisation of k is observed by all parties after the first event and before any trade for sub-period 2. It is not verifiable, so a contract written at inception cannot condition on it.

After an event in sub-period 1 there is a spot market for cover in sub-period 2. Competition among risk-neutral reinsurers sets the price per unit of limit, the rate on line, equal to expected indemnity plus the cost of capital:

$$x = p_2 + k, \qquad \bar{x} = p_2 + \bar{k}. \qquad (1)$$

I write $x_\ell = p_2 + k_\ell$ and $x_h = p_2 + k_h$ for the ends of the support of x. The price of cover after an event exceeds the price of the same cover absent an event, $p_2 + \kappa$, by the factor $1 + \pi$ with $\pi = (k - \kappa)/(p_2 + \kappa)$. Post-event scarcity corresponds to $\bar{k} > \kappa$, but no result below requires this inequality except where stated.

4.2 Contracts and pricing

At inception the cedent buys an occurrence cover with limit L together with a restoration contract (r, g).

Restoration. If an event occurs in sub-period 1, the reinsurer pays L and restores an amount $r \in [0, L]$ of the limit for sub-period 2. Restoration is automatic and binds both parties. An event in sub-period 2 that follows an event in sub-period 1 is indemnified up to r.

Premiums. The cedent pays P at inception and a reinstatement premium $g \geq 0$ after an event in sub-period 1, by offset against the recovery. A negative g would be a payment to the cedent in the event state. It would be an additional limit, which has to be backed by capital like any other, and is therefore not treated as a separate instrument.

Top-up. After an event in sub-period 1 the cedent may buy additional cover $t \in [0, L - r]$ for sub-period 2 at the spot rate x.

If no event occurs in sub-period 1, the original limit remains in force for sub-period 2 and nothing is restored.

The reinsurer's expected outlay consists of the following items. It pays L with probability p_1 in sub-period 1. With probability $1 - p_1$ no event has occurred and it pays L with probability p_2 in sub-period 2. It carries capital L in sub-period 1 at cost κL, and again in sub-period 2 if no event has occurred, at expected cost $(1 - p_1)\kappa L$. After an event it carries capital r at cost kr and pays r with probability p_2. Zero expected profit therefore requires

$$P + p_1 g = P_a + p_1 \bar{x}\, r, \qquad P_a = [p_1 + (1 - p_1)p_2]\, L + \kappa(2 - p_1)L. \tag{2}$$

P_a is the competitive premium of the limit without restoration. Committed restoration is priced at the expected spot rate $\bar{x}$, because the reinsurer that restores the limit forgoes the use of the same capital in the spot market.

The family (r, g) contains the contracts of interest as special cases. The pair $(0, 0)$ is the bare limit, after which the cedent buys at spot or goes uncovered. The pair $(r, \bar{x}r)$ is a *pure forward*: the cedent pays for restored capacity when it is delivered, at the expected spot rate, and $P = P_a$. The pair $(r, 0)$ is a *prepaid restoration*. The pair $(L, \rho P)$ is the *standard clause* with one full reinstatement at rate ρ. An aggregate cover with limit $2L$ pays at most L in each sub-period. Under the capital rule above it requires capital L in each sub-period and has the cash flows of $(L, 0)$. It is the prepaid standard clause.

Figure 1 summarises the sequence of events.

4.3 The cedent's problem

Write $m = L - r$ for the exposure that the contract leaves open after a first event. At spot rate x the cedent leaves an amount $u \in [0, m]$ uninsured and buys $m - u$. Its terminal funds are $W_0 - P$ if no event occurs in sub-period 1, and

$$W_0 - P - g - x(m - u) - u \cdot \mathbf{1}\{\text{event in sub-period 2}\}$$

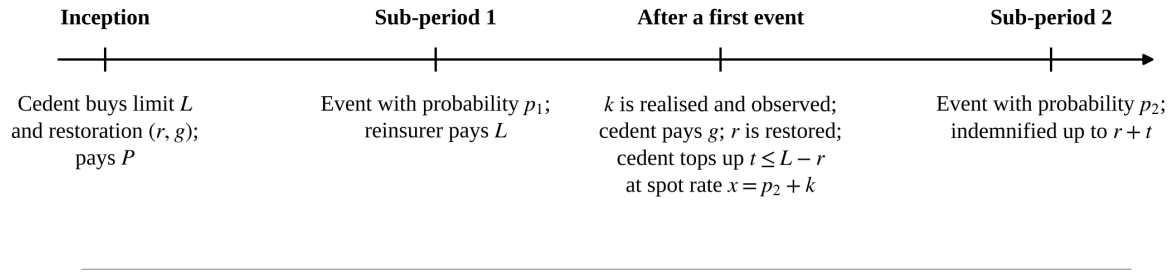


Figure 1. Timing. The contract (r, g) is written at inception. The cost of capital k is realised after a first event and before any trade for sub-period 2.

otherwise. Define the certainty-equivalent cost of the open exposure,

$$c(m, x) = \frac{1}{a} \ln \min_{0 \le u \le m} e^{ax(m-u)} \left[1 - p_2 + p_2 e^{au}\right]. \qquad (3)$$

Using (2) to eliminate P, the cedent's expected utility equals $-\exp\{-a(W_0 - P_a)\} \exp\{\Phi(r, g)\}$, where

$$\Phi(r, g) = a\, p_1 \left(\bar{x}\, r - g\right) + \ln \left[(1 - p_1) + p_1 e^{ag}\, \mathbb{E}\, e^{a\, c(L-r, x)}\right] \qquad (4)$$

and the expectation is over x. The cedent chooses (r, g) to minimise Φ.

4.4 Remarks on the assumptions

The distribution F is the only object that represents post-event scarcity. It is a primitive of the model in the same sense as κ. It can be read as the outcome of market clearing in the spirit of Winter (1994) and Gron (1994): industry capital after the event equals the pre-event stock less a random industry loss, external capital cannot be raised within the period, and k equates the demand for cover with the capacity that remains. The cedent is small and takes k as given. The results hold for every F with the stated support.

The assumption that k is not verifiable rules out restoration schedules indexed to the state of the market. Treaties do not contain such indexation, and the price of capacity for a particular layer after a particular event is not an observable index. Restoration is binding because that is how the clause operates in practice; Section 5.6 considers a restoration that the cedent may decline. The capital rule charges each sub-period for the largest indemnity payable in it. A seller that cannot raise capital after the event, because its obligations are confined to assets in trust, is considered in Section 5.5.

Table 1 collects the notation, including the objects defined in Section 5.1.

Table 1. Notation.

Symbol	Meaning
L	Limit of the layer and size of an event loss
$p_1,\ p_2$	Probability of an event in sub-period 1 and in sub-period 2
a	Coefficient of absolute risk aversion of the cedent
κ	Cost of capital per unit and sub-period, absent a prior event
$k,\ \bar{k}$	Cost of capital in sub-period 2 after an event in sub-period 1, and its mean
$x,\ \bar{x}$	Spot rate on line after an event, $x = p_2 + k$, and its mean
$x_\ell,\ x_h$	Lowest and highest possible spot rate
r	Restoration committed at inception, $0 \le r \le L$
g	Reinstatement premium, paid after a first event
$P,\ P_a$	Premium at inception with and without restoration
m	Exposure left open by the contract after a first event, $m = L - r$
$c(m, x)$	Certainty-equivalent cost of the open exposure m at spot rate x
$D(x)$	Amount the cedent leaves uninsured at spot rate x
$q^*(x)$	Spot demand of a cedent with no restoration, $\max\{0, L - D(x)\}$
$R(m)$	Certainty-equivalent cost of bearing an exposure m without cover
x_0	Choke price, $x_0 = R'(L)$: the spot rate above which a cedent with no restoration buys nothing
$\Phi,\ \psi$	The cedent's objective in general and under the pure forward

5. Results

5.1 Spot demand after a first event

Define

$$D(x) = \frac{1}{a} \ln \frac{(1 - p_2)\, x}{p_2\, (1 - x)}, \qquad R(m) = \frac{1}{a} \ln \left(1 - p_2 + p_2 e^{am}\right),$$

$$\mathrm{KL}(x \,\|\, p_2) = x \ln \frac{x}{p_2} + (1 - x) \ln \frac{1 - x}{1 - p_2}. \qquad (5)$$

$D(x)$ is the amount the cedent wishes to leave uninsured at rate x. $R(m)$ is the certainty-equivalent cost of bearing an exposure m without cover. KL is the relative entropy of an event probability x with respect to p_2.

Lemma 1. For every $m \in [0, L]$ and $x \in (p_2, 1)$:

(i) the cedent leaves $\min\{D(x), m\}$ uninsured, and $D(x) > 0$, so cover bought at spot never restores the full exposure;

(ii) the cost of the open exposure is

$$c(m,x) = \begin{cases} x\,m - \dfrac{1}{a}\,\mathrm{KL}(x \,\|\, p_2) & \text{if } D(x) \le m, \\ R(m) & \text{if } D(x) > m; \end{cases} \tag{6}$$

(iii) c is non-negative, convex and continuously differentiable in m, with $c(0,x) = 0$ and $\partial c/\partial m = \min\{x, R'(m)\}$, where $R'(m) = p_2 e^{am}/(1 - p_2 + p_2 e^{am})$;

(iv) c is non-decreasing in x, with $\partial c/\partial x = m - D(x) > 0$ where $D(x) < m$ and $\partial c/\partial x = 0$ where $D(x) > m$.

Part (i) is Mossin's (1968) result in the present setting. The spot price always contains a cost of capital, so the cedent retains part of the exposure, and it retains more when capital is dearer because D is increasing. Part (iii) yields a choke price. A cedent with open exposure m buys at spot if and only if $x < R'(m)$. I write $x_0 = R'(L)$ for the choke price of a cedent with no restoration and

$$q^*(x) = \max\{0,\, L - D(x)\}$$

for its spot demand. The closed form in part (ii) has a simple reading: a cedent that buys at spot pays the rate x on its whole exposure and recovers a surplus $\mathrm{KL}(x \,\|\, p_2)/a$ from choosing the amount.

Example. Let $L = 1$, $p_1 = p_2 = 0.10$ and $a = 4$, and let the cost of capital after an event be 0.03 or 0.09 with equal probability. The spot rate on line is then 0.13 or 0.19, against an expected indemnity of 0.10, and its mean is $\bar{x} = 0.16$. At the low rate the cedent leaves $D(0.13) = 0.074$ of the limit uninsured and buys 0.926. At the high rate it leaves $D(0.19) = 0.187$ uninsured and buys 0.813. The choke price is $x_0 = 0.858$, far above both rates, so the cedent buys at spot in either state, and in neither does it buy the whole limit. The example is continued after Propositions 1 and 2.

5.2 The reinstatement as a contingent forward

Consider the pure forward, $g = \bar{x}r$. The cedent pays nothing extra at inception and pays the expected spot price for restored capacity if and when an event occurs. No premium is moved across states, so the contract isolates the effect of fixing the price. With $g = \bar{x}r$, minimising (4) is equivalent to minimising

$$\psi(r) = a\,\bar{x}\,r + \ln \mathbb{E}\, e^{a\,c(L-r,x)}. \tag{7}$$

Proposition 1 (contingent forward).

(i) ψ is convex, so the set of optimal restorations is a closed interval.

(ii) Every optimal restoration satisfies $r^* \le q^*(x_\ell) < L$. In particular $r^* = 0$ if $x_\ell \ge x_0$.

(iii) If k is non-degenerate and $x_h < x_0$, every optimal restoration satisfies $r^* > q^*(x_h) > 0$.

(iv) If k is degenerate at a value with $x < x_0$, then ψ is constant on $[0, q^*(x)]$ and strictly increasing on $[q^*(x), L]$.

(v) In general, $r^* = 0$ is optimal if and only if

$$\frac{\mathbb{E}\left[e^{a\,c(L,x)}\min\{x, x_0\}\right]}{\mathbb{E}\,e^{a\,c(L,x)}} \leq \bar{x}. \qquad (8)$$

Part (iv) is the benchmark. When the post-event price is known, a forward at that price for any amount up to what the cedent would buy anyway reproduces the outcome of spot repurchase state by state. The commitment adds nothing, and committing more than the spot demand is costly because the excess cannot be resold. This is true however high the known price is. A high post-event price does not by itself create a demand for restoration.

Part (iii) gives the value of the commitment. When the price is uncertain, the cedent's marginal utility is higher in the states in which capacity is dear, because it pays more for the cover it buys. A forward pays off in those states. The first units of committed restoration, which the cedent would have bought in every state, are therefore a pure hedge and are always taken. The reinstatement is in this precise sense a forward on post-loss capacity, conditional on a loss having occurred.

Parts (ii) and (iii) together locate the optimum. The cedent commits strictly more than it would buy in the dearest market and no more than it would buy in the cheapest. When capacity turns out cheap it tops up at spot. When capacity turns out dear it holds more cover than it would have chosen at that price. The bound $r^* < L$ reflects the cost of capital: a restoration of the whole limit is never chosen when the cedent can set the amount. If capacity after an event is always dearer than the choke price, no restoration is committed. Condition (8) covers the intermediate case in which the cedent would buy at spot in some states only.

Example, continued. Proposition 1 places the forward optimum between 0.813 and 0.926. Numerically it is $r^* = 0.814$. The cedent commits almost exactly what it would buy in the dear market and no more. If capacity turns out cheap it tops up by 0.112, to the 0.926 it would have bought anyway. If capacity turns out dear it buys nothing further. It then pays 0.16 per unit for cover that costs 0.19 at spot, and this gain in the state in which it is poorer is what the forward provides. The certainty-equivalent value of the commitment is 0.0013 of the limit. If the spot rate were 0.16 with certainty, the cedent would buy 0.865 at spot and a forward for any amount up to 0.865 would leave it exactly as well off.

5.3 Prepayment and premium smoothing

Now let the cedent choose how the restoration is paid for. Write $M(r) = \mathbb{E}\,e^{a\,c(L-r,x)}$ and $\Phi_P(r) = \Phi(r, 0)$ for the prepaid contract.

Proposition 2 (prepayment).

(i) For every r, $\Phi(r, g)$ is non-decreasing in g on $g \geq 0$, and strictly increasing unless $r = L$ and $g = 0$. The cedent's preferred reinstatement premium is $g = 0$.

(ii) Φ_P is convex and every optimal prepaid restoration satisfies $r_P < L$.

(iii) Every optimal prepaid restoration is at least as large as the smallest optimal restoration under the pure forward. If the largest optimal forward restoration is positive, every optimal prepaid restoration is strictly larger.

(iv) If $x_h < x_0$, then $r_P > 0$, including when k is degenerate.

Prepayment moves the cost of restored capacity out of the event state, in which the cedent also bears the cost of its remaining exposure, and spreads it over all states. This is a second motive for the clause and it is distinct from the first. It operates when the post-event price is known, and it raises the committed amount above the forward optimum because each unit of prepaid restoration both hedges the price and transfers funds to the state in which they are worth more. The clause as it is written in practice bundles the two: a commitment on quantity and price, and a division of the payment between inception and the event.

Example, continued. With prepayment the optimal restoration rises from 0.814 to $r_P = 0.849$, and the certainty-equivalent gain over spot repurchase rises from 0.0013 to 0.0052 of the limit. Most of the value of the clause in this example comes from smoothing the premium and the smaller part from fixing the price.

Part (i) is the negative result. A reinstatement premium is a payment in the state in which the cedent's funds are most valuable to it, and the competitive reinsurer is indifferent to the timing. The cedent therefore prefers to prepay in full. The model does not generate a positive reinstatement premium, and so does not explain the convention of 100 per cent. Section 7 returns to this.

5.4 The standard clause and spot repurchase

Market practice offers the cedent a coarser choice than the model: a layer is written with a full, binding reinstatement or without one. The next result compares the two directly.

Proposition 3 (standard clause against spot repurchase).

(i) The standard clause (L, g) is strictly preferred to spot repurchase $(0, 0)$ if and only if

$$a\, p_1\, (\bar{x} L - g) + \ln\left[(1 - p_1) + p_1 e^{ag}\right] < \ln\left[(1 - p_1) + p_1\, \mathbb{E}\, e^{a\, c(L,x)}\right]. \qquad (9)$$

(ii) The left-hand side of (9) is strictly increasing in g on $g \geq 0$. A higher reinstatement premium shrinks the set of parameters for which the clause is preferred.

(iii) Let $g = 0$ and replace k by $k + \delta$. The advantage of the clause over spot repurchase, the right-hand side of (9) minus the left-hand side, is increasing in δ at $\delta = 0$ if and only if

$$\omega(0)\, \mathbb{E}_{Q_0}\left[q^*(x)\right] > L, \qquad (10)$$

where $\omega(0) = M(0)/[(1-p_1)+p_1 M(0)]$ and Q_0 is the distribution with density $e^{a\,c(L,x)}/M(0)$ with respect to F. If $x_\ell \geq x_0$ the advantage is strictly decreasing in δ.

(iv) Both contracts are weakly dominated by the optimal prepaid restoration of Proposition 2. The standard clause is strictly dominated, and spot repurchase is strictly dominated if $x_h < x_0$.

Neither contract dominates the other. The clause insures the price and smooths the premium, and it forces the cedent to hold the full limit at a price at which it would have held less. Part (iii) shows how the balance moves with scarcity. Dearer post-event capital raises the cost of the clause by the full limit. It raises the cost of spot repurchase only by the amount the cedent would actually buy, weighted by the marginal utility of the event state. When the cedent would buy most of the limit at spot and is sufficiently averse to the event state, scarcity favours the clause. When capacity is dear enough that the cedent would buy little, further scarcity favours the spot market, and once the price exceeds the choke price the cedent is better off without the commitment. The view that a higher post-loss price always strengthens the case for reinstatement holds only when the flexibility of the spot buyer is ignored.

5.5 Comparative statics, layers, and pre-funded capacity

Proposition 4 (comparative statics).

(i) Replace k by $k+\delta$ with $p_2 + k_h + \delta < 1$. Under the pure forward, the smallest and the largest optimal restoration are non-increasing in δ.

(ii) Fix k and suppose $2p_2 + k < 1$. Then $D(p_2 + k)$ is strictly decreasing in p_2. The bounds $q^*(x_h)$ and $q^*(x_\ell)$ of Proposition 1 are therefore non-decreasing in the event probability of the layer, and strictly increasing where positive.

A uniform rise in the cost of post-event capital lowers the committed amount: the hedging motive is unchanged, while every unit of cover is dearer. Part (ii) concerns the cross-section of layers. For a remote layer the expected loss is small relative to the cost of the capital that backs it, so the rate on line is a large multiple of the event probability and the cedent retains more. Both bounds on committed restoration fall as the layer becomes more remote. Section 6 shows numerically that the optimum itself falls. A programme that reinstates its lower layers and leaves its top layers with one restoration or none is restoring part of its total limit, with the part chosen where restoration is worth most.

The remaining comparison concerns the seller. A balance-sheet reinsurer honours a restoration with capital it holds or raises after the event, at cost k. A seller whose obligations are confined to assets in trust cannot do so. It can offer a restoration r only by placing $L + r$ in trust at inception.

Proposition 5 (pre-funded restoration). Suppose capital placed in trust at inception costs κ per unit and per sub-period in every state. A pre-funded restoration r has expected

capital cost $\kappa(1+p_1)r$, against $p_1\bar{k}\,r$ for a contingent restoration, with the same indemnities. Pre-funding is cheaper if and only if

$$\frac{\bar{k}}{\kappa} > \frac{1+p_1}{p_1}. \qquad (11)$$

If capital in trust instead bears its opportunity cost k after an event, pre-funding costs κr more than contingent restoration for all parameter values.

The assumption that capital in trust costs κ in every state is the one most favourable to collateral, since it shields the pre-funded limit from post-event scarcity altogether. Even so, pre-funding requires the second limit to be carried through the first sub-period, in which it is needed with probability zero, in order to save the scarcity premium in a state that occurs with probability p_1. The threshold in (11) is 51 for $p_1 = 0.02$, 11 for $p_1 = 0.10$ and 4.3 for $p_1 = 0.30$. Collateralised restoration is therefore feasible but expensive, and least expensive on high-frequency layers. It is not infeasible.

5.6 Optional restoration and retained losses

Two extensions use the same apparatus. In the first, the cedent may decline the restoration after observing k, in which case it pays no reinstatement premium and buys at spot. In the second, an event in sub-period 1 also causes the cedent a retained loss $\ell \geq 0$, below the attachment point or above the layer, so that the event state is worse for the cedent than the contract alone implies.

Proposition 6 (extensions).

(i) A cedent that may decline a restoration (r, g) accepts it at spot rate x if and only if $g \leq c(L, x) - c(L - r, x)$. The right-hand side is non-decreasing in x. The restoration is accepted in every state if and only if $g \leq c(L, x_\ell) - c(L - r, x_\ell)$. For the standard clause the condition is $g \leq c(L, x_\ell)$.

(ii) With a retained loss ℓ, Lemma 1 and Proposition 1 are unchanged and the cedent's preferred reinstatement premium is still $g = 0$. The smallest and the largest optimal prepaid restoration are non-decreasing in ℓ, and full restoration, $r_P = L$, is optimal if and only if

$$\frac{e^{a\ell}}{(1-p_1) + p_1 e^{a\ell}} \geq \frac{\bar{x}}{p_2}. \qquad (12)$$

Part (i) gives the incentive constraint that an optional clause must satisfy and the corresponding upper bound on the reinstatement premium. Under the pure forward the constraint fails whenever capacity turns out cheaper than expected: for $r \leq q^*(x)$ the right-hand side equals xr, and the cedent declines if $x < \bar{x}$. An optional restoration at the expected spot price is therefore a call option on capacity, which a competitive reinsurer would have to price above the forward. The binding character of the clause is what allows it to be priced at the expected spot rate. Part (ii) shows that event-dependent losses leave the forward motive unchanged under exponential utility

and strengthen the smoothing motive: a cedent that is poorer after an event values prepaid restoration more. Condition (12) compares two ratios. The left-hand side is the marginal utility of funds in the event state relative to its average over states. The right-hand side is the price of restored capacity relative to its expected indemnity. Without retained losses the left-hand side is one and the condition fails, which is Proposition 2(ii). When the event that consumes the layer also depletes the cedent outside it, restoration of the whole limit can be optimal. This is the one configuration in which the model yields the full restoration that treaties specify.

6. Numerical illustration

This section illustrates the results and reports the comparative statics for which no proof is given. It has no role in the propositions. Throughout, $L = 1$, so that amounts are shares of the limit, and k takes two values with equal probability.

Table 2 reports the certainty-equivalent gain of the prepaid standard clause over spot repurchase, the difference between the two sides of (9) divided by a, for $p_1 = p_2 = 0.10$ and $k \in \{\bar{k} - 0.02, \bar{k} + 0.02\}$. The pattern is that of Proposition 3(iii). For low risk aversion the clause is worse than spot repurchase and becomes more so as capital becomes dearer. For high risk aversion the ranking and the direction are reversed. At a reinstatement premium of 100 per cent of the original premium the entries in the last row become -0.0089, -0.0048, $+0.0022$ and $+0.0089$, and every entry in the first two rows is negative.

Table 2. Certainty-equivalent gain of the prepaid standard clause over spot repurchase, as a share of the limit.

Risk aversion a	$\bar{k} = 0.05$	$\bar{k} = 0.15$	$\bar{k} = 0.30$	$\bar{k} = 0.50$
0.5	-0.0023	-0.0121	-0.0271	-0.0471
2.0	$+0.0013$	-0.0005	-0.0095	-0.0290
4.0	$+0.0043$	$+0.0095$	$+0.0183$	$+0.0277$

Note: $p_1 = p_2 = 0.10$, $L = 1$, $\kappa = 0.03$, k equal to $\bar{k} - 0.02$ or $\bar{k} + 0.02$ with equal probability. A positive entry means the clause is preferred. Source: author's calculations.

Figure 2 varies the event probability of the layer, with $p_1 = p_2 = p$, $a = 4$, $\kappa = 0.03$ and $k \in \{0.03, 0.09\}$. Panel (a) shows the optimal restoration under the pure forward and under prepayment, together with the bounds of Proposition 1. Restoration falls from about nine tenths of the limit at $p = 0.30$ to about four tenths at $p = 0.01$. Panel (b) shows the gain of full restoration over spot repurchase. The prepaid clause is preferred for p above about 0.03 and spot repurchase below it, so the most remote layers are better left without a binding restoration. At a rate of 100 per cent the clause is worse than spot repurchase for every p in this example, and the shortfall grows with p, because the original premium, and with it the reinstatement premium, is larger on high-frequency layers.

I also evaluated the optimal restoration on 2,000 random draws of the parameters, with $p_1, p_2 \in [0.02, 0.30]$, $a \in [0.5, 6]$ and two-point distributions of k. In every draw the optimal

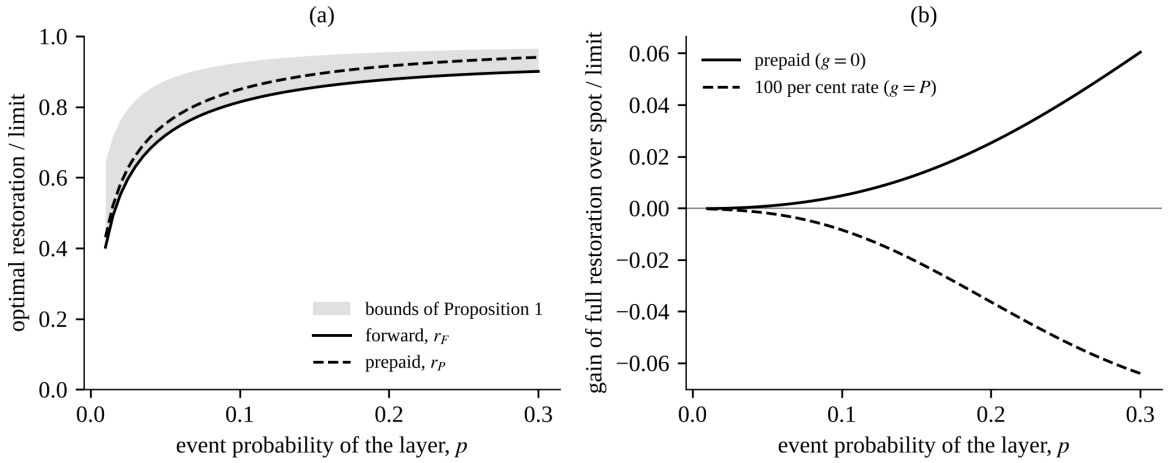


Figure 2. Restoration across layers. Panel (a): optimal restoration as a share of the limit under the pure forward and under prepayment; the shaded band is the interval between the bounds of Proposition 1. Panel (b): certainty-equivalent gain of full restoration over spot repurchase, as a share of the limit, when the restoration is prepaid and when it carries a reinstatement premium equal to the original premium. Parameters: $p_1 = p_2 = p$, $a = 4$, $\kappa = 0.03$, $k \in \{0.03, 0.09\}$ with equal probability. Source: author's calculations.

restoration, forward and prepaid, was non-increasing in a uniform upward shift of k, non-decreasing in the event probability p_2 and non-decreasing in risk aversion. Proposition 4 proves the first of these for the forward and the second for its bounds; the others are numerical findings. The optimal restoration is not monotone in the dispersion of k. A wider spread lowers spot demand in the dearest market, and the committed amount follows it down, although the commitment is worth more. The certainty-equivalent value of the forward increased with dispersion in most draws but not in all, and I make no general claim about it.

7. Discussion

7.1 The institutional record

The model accounts for several features of practice. Restoration is demanded because the price of post-loss capacity is uncertain when the treaty is written, which is the condition documented by the capacity-constraint literature (Froot and O'Connell 1999; Froot 2001). The amount demanded is less than the whole limit and is smaller on remote layers, which matches the decline in the number of reinstatements from working layers to the top of a programme. Layers on which capacity after an event would cost more than the cedent's choke price are left without restoration. Collateralised sellers can supply the provision only by pre-funding it, which is expensive except on high-frequency layers, and such sellers generally write single-shot covers (Artemis 2017).

Two features of practice need more than the baseline model. Treaties restore the consumed limit in full, whereas the baseline cedent chooses a partial restoration. Proposition 6(ii) shows that full restoration is optimal when the event also causes the cedent sufficiently large losses outside the layer, which is the usual situation of a primary insurer after a catastrophe. The model also does not separate a reinstatement from an aggregate cover. With one potential

event per sub-period, an aggregate limit of twice the occurrence limit is the prepaid standard clause, and the model has nothing to say about why the market might price aggregate promises differently. An account of that difference requires a richer loss process, in which the number of events and their dependence matter.

7.2 The reinstatement premium

Proposition 2(i) says that the cedent prefers to prepay. Observed treaties charge for restoration when it is delivered, typically at the original rate. The model therefore leaves the level of the reinstatement premium unexplained, and I do not claim otherwise. Two frictions outside the model could account for it. The first is funding. If a premium received after the event reduces the capital the reinsurer has to commit when capital is dearest, the reinsurer is no longer indifferent to the timing of payment, and a positive reinstatement premium can be jointly optimal. Introducing this channel gives prepaid premium an implicit return as well, and the forward price then differs from the expected spot price, so the benchmark of Proposition 1 is lost. The second is incentives. A reinstatement premium proportional to the amount reinstated is a co-payment on the claim. It may discipline the reporting and allocation of losses, in the way that retentions do in models of moral hazard, but this requires a model in which losses are not publicly observed.

The model does make one statement about the rate. By Proposition 3(ii) a given rate is more costly to the cedent the larger the reinstatement premium it implies, and the premium is larger where the original premium is larger. Figure 2(b) shows the consequence: a rate of 100 per cent is most costly on high-frequency layers. This is consistent with the observation that working layers often carry reinstatements at reduced or no additional premium, although the model does not determine the rate on any layer.

7.3 Ambiguity

The model assumes that event probabilities are known and that only the price of capital is uncertain. A companion paper studies the supply of reinsurance capacity when loss models disagree and shows that a contract exposing the seller to several event draws under a common unknown model carries a load that a single-draw contract does not (Aslan 2026). A reinstated limit is a second draw. Under model ambiguity a restoration would therefore carry the same multi-event load as an aggregate cover, and the load would reduce committed restoration in lines where models disagree. The present model, in which capital is scarce after a loss and probabilities are agreed, isolates a different friction and should be read alongside that one.

7.4 Limitations

Four limitations should be noted. First, the exponential value function is used throughout. Partial cover at a loaded price holds for any concave value function (Mossin 1968), and the covariance argument behind Proposition 1(iii) uses only that marginal utility is higher when capacity is dearer. The closed form (6) and the bounds expressed through D are specific to the

exponential case, and with wealth effects the spot demand would also depend on the premiums already paid. Second, the layer loss is binary. A partial loss to the layer would leave part of the limit in place and would bring in the pro rata rule for the reinstatement premium, which the present model does not address. Third, the distribution of the post-event cost of capital is exogenous. A market equilibrium in which reinstatements written across the industry affect the capacity available after an event, and hence F itself, is a natural continuation. Fourth, the comparison with practice is qualitative. Reinstatement terms by layer are recorded in placement documents but are not available as a public dataset, and the predictions of Propositions 3 and 4 could be tested if they were.

8. Conclusion

This paper has asked what a reinstatement provision is for. In a model in which the cost of reinsurance capital after a loss is uncertain, the provision is a contingent forward on post-loss capacity. Its value comes from the uncertainty of the post-loss price and not from its level. A cedent that can choose the amount commits part of the limit: more than it would buy in the dearest market, no more than it would buy in the cheapest, and less on remote layers. Prepayment adds the smoothing of premium across states. Full, binding restoration can be worse than buying at spot, and it becomes worse when capacity is so dear that the cedent would buy little. Collateralised sellers can restore a limit only by pre-funding it, at a cost that is prohibitive except where events are frequent.

The analysis also marks the boundary of the explanation. Risk sharing under costly capital implies that restoration should be prepaid. The reinstatement premium that treaties in fact charge on delivery must reflect something else, and the funding of post-event capacity and the incentives of the cedent in reporting losses are the candidates. Both can be studied within the framework set out here, and each would yield predictions about how reinstatement rates vary with the state of the market and across layers.

Appendix: Proofs

Preliminaries. If no event occurs in sub-period 1, the original limit covers sub-period 2 and the cedent's funds are $W_0 - P$. If an event occurs, the cedent pays g, faces an open exposure $m = L - r$ and chooses $u \in [0, m]$. Its expected utility conditional on x is $-\exp\{-a(W_0 - P - g)\}\, e^{ax(m-u)}\,[1 - p_2 + p_2 e^{au}]$, which is maximised by minimising the last two factors. By (3) the maximised value is $-\exp\{-a(W_0 - P - g)\}e^{a\,c(m,x)}$. Unconditional expected utility is therefore $-e^{-a(W_0-P)}\left[(1-p_1) + p_1 e^{ag}\,\mathbb{E}\,e^{a\,c(L-r,x)}\right]$. Substituting $P = P_a + p_1\bar{x}r - p_1 g$ from (2) gives (4).

Proof of Lemma 1. The function minimised in (3) is $e^{axm}h(u)$ with $h(u) = (1-p_2)e^{-axu} + p_2 e^{a(1-x)u}$. Since $0 < x < 1$, h is a sum of two exponentials with rates of opposite sign and is strictly convex. Its derivative vanishes where $p_2(1-x)e^{a(1-x)u} = (1-p_2)xe^{-axu}$, that is, where $e^{au} = (1-p_2)x/[p_2(1-x)]$, or $u = D(x)$. Because $x > p_2$, $D(x) > 0$. By strict convexity the minimiser on $[0, m]$ is $\min\{D(x), m\}$. This proves (i).

Part (ii). If $D(x) \le m$, set $u = D(x)$. Then $p_2 e^{aD} = (1-p_2)x/(1-x)$, so $1 - p_2 + p_2 e^{aD} = (1-p_2)/(1-x)$, and

$$a\,c(m,x) = axm - axD(x) + \ln\frac{1-p_2}{1-x} = axm - x\ln\frac{x}{p_2} - (1-x)\ln\frac{1-x}{1-p_2},$$

where the second equality uses $axD(x) = x\ln(x/p_2) + x\ln[(1-p_2)/(1-x)]$. If $D(x) > m$ the minimiser is $u = m$ and (3) gives $c(m,x) = R(m)$.

Part (iii). On the first branch $\partial c/\partial m = x$ and on the second $\partial c/\partial m = R'(m)$. The inequality $R'(m) < x$ is equivalent to $p_2 e^{am}(1-x) < x(1-p_2)$, that is, to $m < D(x)$. Hence $\partial c/\partial m = \min\{x, R'(m)\}$, and the two expressions coincide at $m = D(x)$, so the derivative is continuous. It is non-decreasing in m because R' is increasing, so c is convex in m. Since $D(x) > 0$, $c(0,x) = R(0) = 0$, and since the derivative is positive, $c \ge 0$ with equality only at $m = 0$.

Part (iv). On the first branch, $\partial c/\partial x = m - \frac{1}{a}\ln\frac{x(1-p_2)}{(1-x)p_2} = m - D(x)$. On the second branch c does not depend on x. ■

Proof of Proposition 1. *Part (i).* By Lemma 1(iii), $c(L-r,x)$ is convex in r for each x. For convex functions f_x and $\lambda \in [0,1]$, convexity and Hölder's inequality give $\mathbb{E}\, e^{f_x(\lambda r + (1-\lambda)r')} \le \mathbb{E}\left[e^{\lambda f_x(r)} e^{(1-\lambda)f_x(r')}\right] \le \left(\mathbb{E}\, e^{f_x(r)}\right)^{\lambda}\left(\mathbb{E}\, e^{f_x(r')}\right)^{1-\lambda}$, so $\ln \mathbb{E}\, e^{a\,c(L-r,x)}$ is convex in r. The first term of (7) is linear. Hence ψ is convex and continuous on $[0,L]$ and its minimisers form a closed interval.

The integrand $e^{a\,c(L-r,x)}$ is continuously differentiable in r with a derivative bounded uniformly in x, so differentiation under the expectation is permitted and

$$\psi'(r) = a\left[\bar{x} - \mathbb{E}_{Q_r}\min\{x, R'(L-r)\}\right], \qquad \frac{dQ_r}{dF} = \frac{e^{a\,c(L-r,x)}}{\mathbb{E}\, e^{a\,c(L-r,x)}}. \tag{A1}$$

Part (ii). Let $r > L - D(x_\ell)$ and $m = L - r$. Since D is increasing, $m < D(x_\ell) \le D(x)$ for every x in the support. By Lemma 1(iii) the marginal cost equals $R'(m)$ for every such x, and $R'(m) < x_\ell \le \bar{x}$ because $m < D(x_\ell)$. Hence $\psi'(r) > 0$ for all $r \in (\max\{0, L - D(x_\ell)\}, L]$, and every minimiser is at most $\max\{0, L - D(x_\ell)\} = q^*(x_\ell)$, which is below L because $D(x_\ell) > 0$. If $x_\ell \ge x_0 = R'(L)$, then $D(x_\ell) \ge L$ and the bound is zero.

Part (iii). The condition $x_h < x_0$ is equivalent to $D(x_h) < L$, so $q^*(x_h) = L - D(x_h) > 0$. Let $r \le q^*(x_h)$, so that $m \ge D(x_h) \ge D(x)$ on the support. The marginal cost equals x and (A1) becomes

$$\psi'(r) = -a\,\frac{\mathrm{Cov}\left(x,\, e^{a\,c(m,x)}\right)}{\mathbb{E}\, e^{a\,c(m,x)}}.$$

By Lemma 1(iv), $\partial c/\partial x = m - D(x) > 0$ for every $x < x_h$ in the support, so $c(m,\cdot)$ is strictly increasing on the support. The covariance of a non-degenerate random variable with a strictly increasing function of itself is strictly positive. Hence $\psi' < 0$ on $[0, q^*(x_h)]$, and by convexity every minimiser exceeds $q^*(x_h)$.

Part (iv). Let x be the single value. For $r \le q^*(x)$ we have $m \ge D(x)$ and, by (6),

$\psi(r) = axr + ax(L-r) - \mathrm{KL}(x \,\|\, p_2) = axL - \mathrm{KL}(x \,\|\, p_2)$, which does not depend on r. For $r > q^*(x)$ the marginal cost is $R'(L-r) < x$ and $\psi'(r) = a[x - R'(L-r)] > 0$.

Part (v). By convexity, $r = 0$ minimises ψ on $[0, L]$ if and only if $\psi'(0) \geq 0$. By (A1) with $R'(L) = x_0$ this is (8). ■

Proof of Proposition 2. *Part (i).* Differentiating (4),

$$\frac{\partial \Phi}{\partial g} = -a\,p_1 + \frac{a\,p_1 e^{ag} M(r)}{(1-p_1) + p_1 e^{ag} M(r)} = a\,p_1(1-p_1)\,\frac{e^{ag}M(r) - 1}{(1-p_1) + p_1 e^{ag} M(r)}.$$

By Lemma 1(iii), $c \geq 0$ with equality only at $m = 0$, so $M(r) \geq 1$ with equality only at $r = L$. For $g \geq 0$ the numerator is therefore non-negative and vanishes only if $g = 0$ and $r = L$.

Part (ii). $\Phi_P(r) = a\,p_1 \bar{x}\, r + \ln[(1-p_1) + p_1 M(r)]$. The argument of the logarithm is the expectation of e^f over a distribution that puts mass $1 - p_1$ on the function $f \equiv 0$ and mass p_1 on the functions $a\,c(L-r, x)$, $x \sim F$. All of these are convex in r, so the logarithm is convex by the inequality used in the proof of Proposition 1(i). Differentiating,

$$\Phi_P'(r) = a\,p_1 \left[\bar{x} - \omega(r)\,\mathbb{E}_{Q_r} \min\{x, R'(L-r)\}\right], \qquad \omega(r) = \frac{M(r)}{(1-p_1) + p_1 M(r)}. \tag{A2}$$

Since $M(r) \geq 1$, $\omega(r) \geq 1$, with equality only at $r = L$. At $r = L$, $m = 0$, $\omega = 1$ and the marginal cost is $\min\{x, R'(0)\} = p_2$, because $R'(0) = p_2 < x$. Hence $\Phi_P'(L) = a\,p_1(\bar{x} - p_2) = a\,p_1 \bar{k} > 0$, and by continuity of Φ_P' no minimiser equals L.

Part (iii). From (A1) and (A2), $\Phi_P'(r) - p_1 \psi'(r) = -a\,p_1[\omega(r) - 1]\,\mathbb{E}_{Q_r} \min\{x, R'(L-r)\}$, which is negative for $r < L$ because the marginal cost is positive. Let $\underline{r}$ be the smallest minimiser of ψ. For $r < \underline{r}$, $\psi'(r) < 0$, so $\Phi_P'(r) < 0$ and r does not minimise Φ_P. Let $\bar{r}$ be the largest minimiser of ψ and suppose $\bar{r} > 0$. By Proposition 1(ii), $\bar{r} < L$, so $\bar{r}$ is interior, $\psi'(\bar{r}) = 0$ and $\Phi_P'(\bar{r}) < 0$. By convexity of Φ_P, every minimiser of Φ_P exceeds $\bar{r}$.

Part (iv). If $x_h < x_0$, then $D(x) < L$ on the support, the marginal cost at $r = 0$ is x, and $c(L, x) > 0$, so $M(0) > 1$ and $\omega(0) > 1$. Since $c(L, \cdot)$ is non-decreasing, $\mathbb{E}_{Q_0} x \geq \bar{x}$. By (A2), $\Phi_P'(0) \leq a\,p_1 \bar{x}[1 - \omega(0)] < 0$. ■

Proof of Proposition 3. *Part (i).* By Lemma 1(iii), $c(0, x) = 0$, so $\Phi(L, g) = a\,p_1(\bar{x}L - g) + \ln[(1-p_1) + p_1 e^{ag}]$, while $\Phi(0, 0) = \ln[(1-p_1) + p_1 M(0)]$. The cedent prefers the contract with the smaller value of Φ.

Part (ii). This is Proposition 2(i) at $r = L$: for $g > 0$ the derivative of $\Phi(L, g)$ in g is strictly positive.

Part (iii). With $g = 0$ the left-hand side of (9) is $a\,p_1 \bar{x} L$, and its derivative in δ is $a\,p_1 L$. By Lemma 1(iv), $\partial c(L, x)/\partial x = \max\{0, L - D(x)\} = q^*(x)$ for $x \neq x_0$; $c(L, \cdot)$ is Lipschitz, so differentiation under the expectation is permitted. The derivative of the right-hand side at $\delta = 0$ is

$$\frac{a\,p_1\,\mathbb{E}\left[e^{a\,c(L,x)} q^*(x)\right]}{(1-p_1) + p_1 M(0)} = a\,p_1\,\omega(0)\,\mathbb{E}_{Q_0}\left[q^*(x)\right].$$

The advantage of the clause rises if and only if this exceeds $a\,p_1 L$, which is (10). If $x_\ell \geq x_0$ then $q^*(x) = 0$ on the support, the right-hand side of (9) does not vary with $\delta \geq 0$, and the advantage falls at rate $a\,p_1 L$.

Part (iv). Both contracts belong to the family over which Φ is minimised. By Proposition 2(i), $\Phi(L, g) \geq \Phi(L, 0)$, and by Proposition 2(ii), $\Phi(L, 0) > \min_r \Phi_P(r)$. By Proposition 2(iv), $\Phi(0, 0) > \min_r \Phi_P(r)$ if $x_h < x_0$. ■

Proof of Proposition 4. *Part (i).* Write $\psi(r; \delta)$ for (7) when each realisation x is replaced by $x + \delta$, and let $m = L - r$, $s(x) = \max\{0, m - D(x + \delta)\}$ and $\mu(x) = \min\{x + \delta, R'(m)\}$. By (A1), $\psi'(r; \delta) = a[\bar{x} + \delta - N/M]$ with $N = \mathbb{E}[e^{a\,c(m, x+\delta)}\mu(x)]$ and $M = \mathbb{E}\, e^{a\,c(m, x+\delta)}$. By Lemma 1, $c(m, x + \delta)$ and $\mu(x)$ are Lipschitz in δ, with derivatives $s(x)$ and $\mathbf{1}\{x + \delta < R'(m)\}$ respectively wherever $x + \delta \neq R'(m)$. Hence N/M is absolutely continuous in δ and, for almost every δ,

$$\frac{\partial}{\partial \delta}\frac{N}{M} = Q\{x + \delta < R'(m)\} + a\,\mathrm{Cov}_Q\left(s(x), \mu(x)\right),$$

where Q has density $e^{a\,c(m, x+\delta)}/M$ with respect to F. The function s is non-increasing in x and μ is non-decreasing in x, so the covariance is non-positive (Chebyshev's sum inequality). The first term is at most one. Therefore $\partial\psi'(r; \delta)/\partial\delta \geq a\,[1 - Q\{x + \delta < R'(m)\}] \geq 0$ for almost every δ, and $\psi'(r; \delta)$ is non-decreasing in δ for every r.

Let $\delta' > \delta$ and let $\bar{r}(\delta)$ be the largest minimiser at δ. For $r > \bar{r}(\delta)$, $\psi'(r; \delta) > 0$, hence $\psi'(r; \delta') > 0$ and, by convexity in r, no minimiser at δ' exceeds $\bar{r}(\delta)$. Let $\underline{r}(\delta')$ be the smallest minimiser at δ'. For $r < \underline{r}(\delta')$, $\psi'(r; \delta') < 0$, hence $\psi'(r; \delta) < 0$ and no minimiser at δ is below $\underline{r}(\delta')$.

Part (ii). $aD(p_2 + k) = \ln(1 - p_2) + \ln(p_2 + k) - \ln p_2 - \ln(1 - p_2 - k)$. Its derivative in p_2 is

$$\frac{1}{p_2 + k} - \frac{1}{p_2} - \frac{1}{1 - p_2} + \frac{1}{1 - p_2 - k} = k\left[\frac{1}{(1 - p_2)(1 - p_2 - k)} - \frac{1}{p_2(p_2 + k)}\right],$$

which is negative if and only if $p_2(p_2 + k) < (1 - p_2)(1 - p_2 - k)$. Expanding, this is $0 < 1 - 2p_2 - k$. The statement about the bounds follows from $q^*(x) = \max\{0, L - D(x)\}$. ■

Proof of Proposition 5. A pre-funded amount r is held in trust throughout sub-period 1, at cost κr. If no event occurs in sub-period 1 it is released, because at most one event can follow and the original limit covers it. If an event occurs, which has probability p_1, it is held through sub-period 2 at cost κr under the first assumption and kr under the second. The expected capital cost is $\kappa(1 + p_1)r$ in the first case and $\kappa r + p_1\bar{k}r$ in the second. A contingent restoration is backed only in sub-period 2 after an event, at expected cost $p_1\bar{k}r$. Expected indemnities are $p_1 p_2 r$ under both arrangements. Comparing $\kappa(1 + p_1)r$ with $p_1\bar{k}r$ gives (11). ■

Proof of Proposition 6. *Part (i).* After an event at spot rate x, a cedent that accepts pays g and faces open exposure $L - r$; by the Preliminaries its certainty-equivalent funds are $W_0 - P - g - c(L - r, x)$. A cedent that declines faces open exposure L and has certainty-equivalent funds $W_0 - P - c(L, x)$. It accepts if and only if $g \leq c(L, x) - c(L - r, x)$. By Lemma 1(iv) the derivative of the right-hand side in x is $\max\{0, L - D(x)\} - \max\{0, L - r - D(x)\} \geq 0$.

Acceptance in every state is therefore equivalent to acceptance at x_ℓ. For $r = L$, $c(0, x_\ell) = 0$.

Part (ii). With a retained loss ℓ in the event state, funds after an event are lower by ℓ for every choice of the cedent, and the event-state term in the Preliminaries is multiplied by $e^{a\ell}$. The decision problem after the event is unchanged, so Lemma 1 holds. Expression (4) becomes $\Phi_\ell(r, g) = a\,p_1(\bar{x}r - g) + \ln[(1 - p_1) + p_1 e^{a(g+\ell)} M(r)]$. Under the pure forward, $\Phi_\ell(r, \bar{x}r) = \ln[(1-p_1)+p_1 e^{a\ell} e^{\psi(r)}]$, an increasing function of $\psi(r)$, so Proposition 1 is unchanged. The derivative of Φ_ℓ in g has the sign of $e^{a(g+\ell)} M(r) - 1$, which is positive for $g \geq 0$ and $\ell > 0$, so the preferred reinstatement premium is $g = 0$. The prepaid objective $\Phi_\ell(r, 0)$ is convex in r by the argument in the proof of Proposition 2(ii), and its derivative is given by (A2) with $\omega_\ell(r) = e^{a\ell} M(r)/[(1 - p_1) + p_1 e^{a\ell} M(r)]$ in place of $\omega(r)$. Since $\omega_\ell(r)$ is increasing in ℓ, this derivative is non-increasing in ℓ for every r, and the argument at the end of the proof of Proposition 4(i), with the inequalities reversed, shows that the smallest and the largest minimiser are non-decreasing in ℓ. By convexity, $r = L$ is a minimiser if and only if the derivative at L is non-positive. At $r = L$, $M = 1$ and the marginal cost is p_2, so the derivative is $a\,p_1[\bar{x} - \omega_\ell(L)\,p_2]$ with $\omega_\ell(L) = e^{a\ell}/[(1 - p_1) + p_1 e^{a\ell}]$, which gives (12). ■

Declarations

Data availability. The paper uses no data. The code for the numerical illustration is available from the author on request.

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