TOPICS OF APPLIED RISK MANAGEMENT

Interest Rate Risk in Insurance ALM

Replicating portfolios and swaptions

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INTRODUCTION

Why an insurer’s view of interest rate risk

About me

- More than a decade in treaty reinsurance underwriting across EMEA: casualty, motor and liability lines
- PhD in Economics, Université de Fribourg; research on insurance cycles, catastrophe risk and the financialisation of reinsurance
- Lecturer, Kalaidos University of Applied Sciences, Zürich

What you get today

- The bank toolkit you have built, applied to an insurer’s balance sheet
- Two worked cases on the CHF swap curve of 19 October 2026, every number reproducible
- An honest view of where the tools break

INTRODUCTION

Where we pick up

What you have built

- Duration, convexity and DV01
- Key-rate durations; n swaps for n key rates
- EVE versus NII
- Replicating portfolios for non-maturing deposits

What changes for an insurer

1. The duration gap flips sign: a life insurer typically loses value when rates fall.
2. Liabilities contain options written to policyholders: convexity and volatility exposure that swaps cannot hedge.
3. Replicating portfolios become a tool for valuation, capital and hedging, not only for a stable margin.

INTRODUCTION

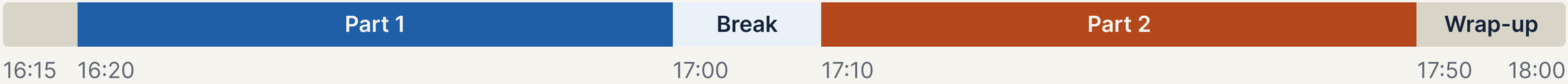
Learning objectives

1. Explain why a life insurer’s interest rate exposure differs from a bank’s, and which policyholder options make it non-linear
2. Describe how replicating portfolios are built for insurance liabilities, from zero-coupon bonds for fixed cash flows to bonds plus swaptions for guarantees, and what insurers use them for
3. Price a European swaption with the Bachelier and Black formulas and interpret its delta and vega
4. Explain why a swap hedge leaves a guarantee’s convexity and vega exposure open, and where insurers’ demand for swaptions comes from

Parts 1 and 2 work through these four objectives in turn.

INTRODUCTION

Plan for today, 16:15 to 18:00



16:15	Opening	Where we pick up; objectives
16:20	Part 1	Replicating portfolios in insurance, from claims reserves to life guarantees (Cases A and B)
17:00	Break	10 minutes
17:10	Part 2	Swaptions: mechanics, pricing, models and calibration, the market, hedging practice
17:50	Wrap-up	Limits, takeaways and your questions

PART 1 · 16:20–17:00

1

Replicating portfolios in insurance

From cash-flow matching for claims reserves to liabilities with options

Two balance sheets, opposite exposures

Bank

Assets	Liabilities
Loans and mortgages Long, fixed rate	**Deposits** Non-maturing, administered rate

Assets longer than liabilities. EVE falls when rates rise.

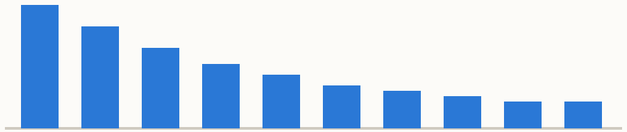
Life insurer

Assets	Liabilities
Bonds, mortgages, real estate Often shorter than the liabilities	**Policy reserves** Savings, annuities, pensions: long-dated, with guarantees

Liabilities longer than assets. EVE falls when rates fall, and the guarantees leave it short convexity.

Non-life insurers sit in between: most claims reserves run off within a few years; long-tail liability lines are the exception.

Three kinds of liability cash flow



1 · Linear

Fixed or predictable

Claims reserves, annuities in payment

Replicate with zero-coupon bonds



2 · Linear, lagged

Administered rates

Profit participation, BVG crediting

Like deposits: credited rates follow smoothed market yields



3 · Non-linear

Options and guarantees

Guaranteed rates, surrender rights, annuity conversion

Replicate with bonds plus options: swaptions

A bank’s deposit model lives in the middle column. Today: the left column (Case A), then the right one (Case B).

Your moving average is an insurer’s book yield

$$X_{i,t} = \frac{1}{i} \sum_{j=0}^{i-1} r_{i,\,t-j}$$

In the deposit model: the yield of rolling i-month tranches.

Insurers hold such ladders: book yields and credited rates lag the market.

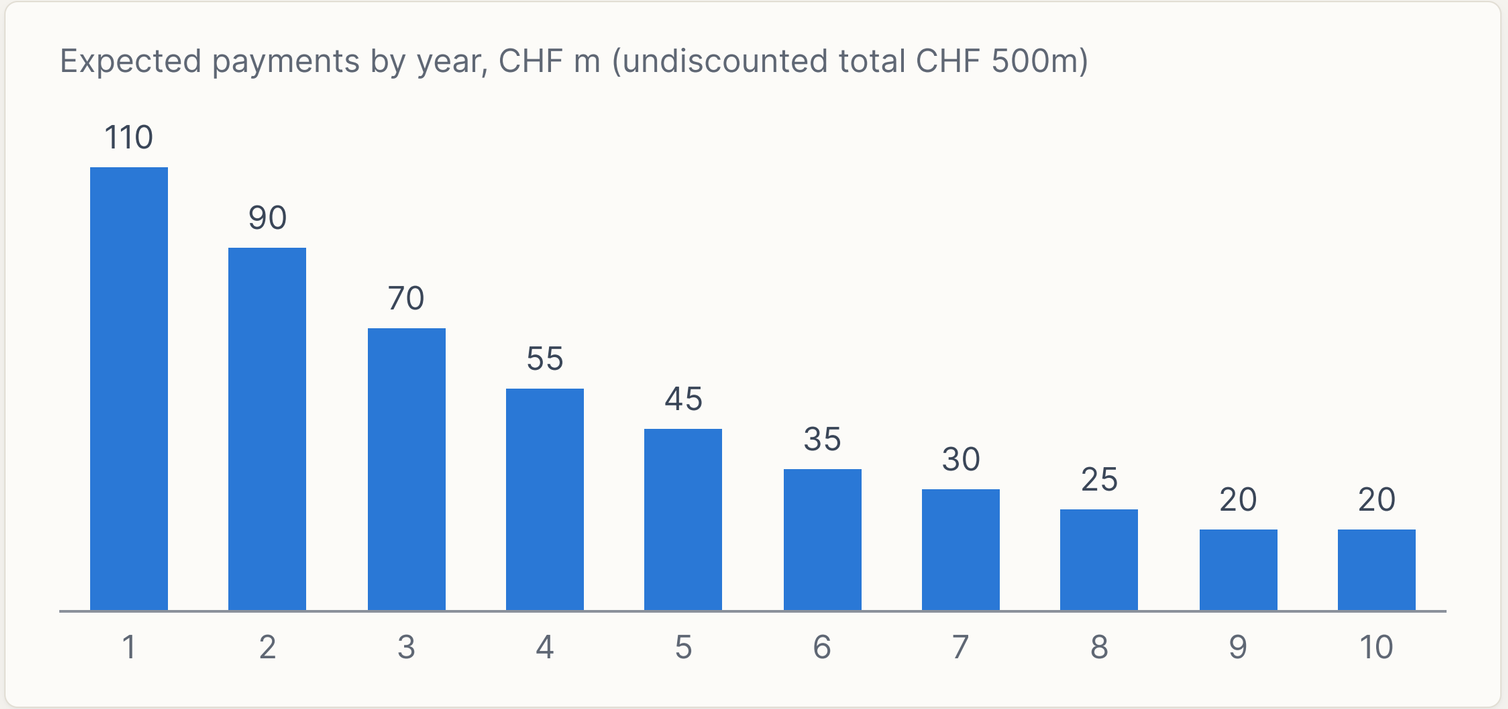
1. Book yield below the guarantee: the insurer pays the gap
2. Market far above book yield: policyholders surrender to reinvest



Stylised rate path; book yield = 10-year average of purchase yields.

CASE A · CASUALTY RESERVE

A claims reserve is a strip of zero-coupon bonds



Best estimate, discounted
CHF 488.8m

Duration
3.8 years

DV01, parallel
CHF 187k per bp

Key-rate DV01 at 2y · 5y · 10y
CHF 50k · 82k · 55k

Face values equal the expected payments; discounting on the CHF SARON curve of 19 October 2026. For long-tail casualty, claims inflation and payment timing often matter more than the rate.

CASE A · CASUALTY RESERVE

Hedging Case A: n swaps for n key rates

Receiver swap	Fixed rate	Notional, CHF m
2 years	0.38%	245.0
5 years	0.60%	165.7
10 years	0.76%	58.6
Total		469.3

Three equations, three unknowns: the swaps' key-rate DV01s times their notionals equal the reserve's key-rate DV01s.

Check: revalue both

Parallel −100bp

Reserve +19.283m, swaps +19.289m: **net +CHF 6k**

Steepener: 2y −50bp, 10y +50bp

Net +CHF 30k

Parallel +100bp

Net +CHF 5k

Linear liabilities need nothing new. What remains is the curve you discount on, claims inflation and payment timing.

Options written to policyholders

Every guarantee is an option the policyholder holds. The insurer is short it.

Policy feature	Bites when	The insurer is short
Minimum crediting rate, reset yearly	rates fall	a strip of floors
Rate guaranteed on future premiums	rates fall	receiver swaptions
Annuity conversion rate (BVG minimum 6.8%)	rates fall	receiver-swaption-like options
Surrender at a guaranteed value	rates rise	payer-swaption-like options

⚠ Equitable Life (UK) sold guaranteed annuity rates when interest rates were high. As rates fell in the 1990s the guarantees went deep into the money; the society closed to new business in 2000.

Replicating a liability with options: the method

1 Scenarios
a market-consistent economic scenario generator, plus stress scenarios

→

2 Liability model
cash flows or values per scenario, with management actions and policyholder behaviour

→

3 Candidate instruments
zero-coupon bonds; swaptions by expiry, tenor and strike; equity options

→

4 Fit
least squares across scenarios

→

5 Validate
out of sample and under stress; compare values; refit

The fitting problem

$$\min_{w} \sum_{s=1}^{N} \left(L^{(s)} - \sum_{k} w_k \, c_k^{(s)} \right)^2$$

L in scenario s: the liability's payoff. c for instrument k: that instrument's payoff in the same scenario. w for instrument k: its position.

Cash-flow matching fits every payment date; terminal-value matching values everything at one horizon.

Same idea, different problem

	Bank: non-maturing deposits	Insurer: liabilities with options
Target	Client rate Y(t)	Liability payoffs per scenario
Regressors	Moving averages of market rates	Payoffs of candidate instruments
Data	Historical time series	Simulated scenarios from the insurer's own models
Constraints	Weights ≥ 0, summing to 1	Usually free; stability constraints in practice
Output	Tenor mix of rolling tranches: a stable margin	Bonds plus options: value, capital, hedge

Same least squares, different data, different purpose.

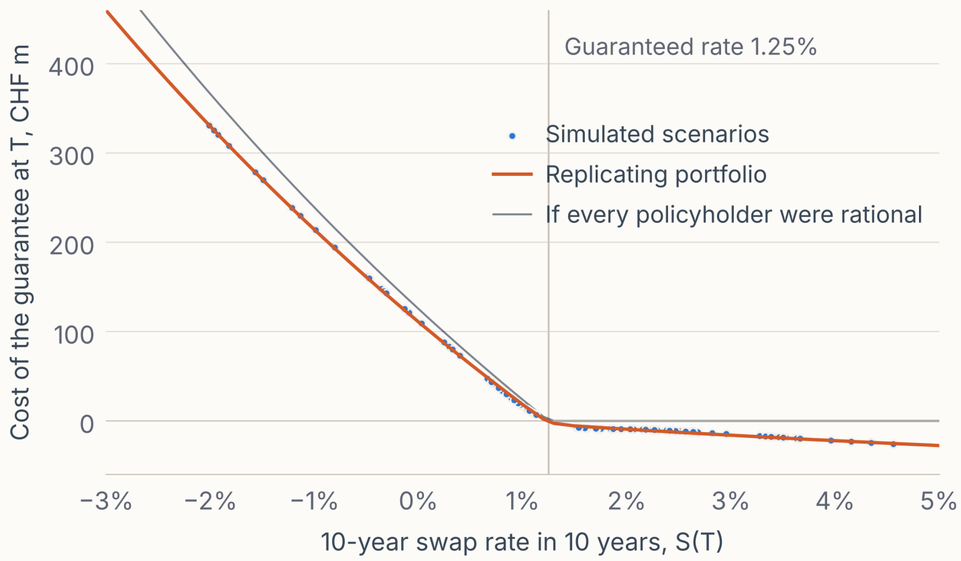
CASE B · A GUARANTEE

What the regression finds

CHF 1,000m block; in 10 years, extend 10 more years at a guaranteed 1.25% (forward swap rate today: 1.01%).

Instrument (all 10y × 10y)	CHF m
Zero-coupon bond, 10y	−3.7
Forward receiver swap, 1.25%	82.8
Receiver, strike 1.00%	56.2
Receiver, strike 1.25%	694.3
Receiver, strike 1.50%	76.1

R^2, in and out	Value	If all rational
0.9999	**CHF 62.8m**	**CHF 77.7m**



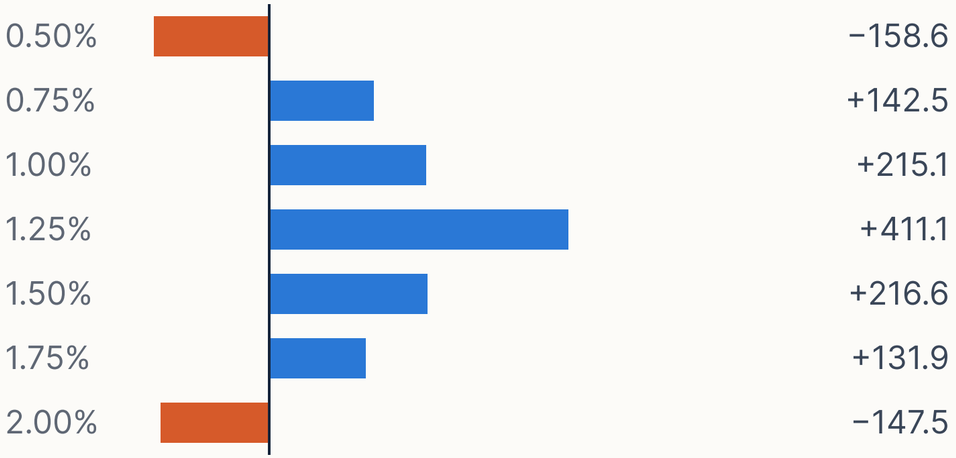
Assumed: 90% extend deep in the money, 10% even out of it. Recovered: CHF 827m of receivers (about the 80% who switch) and a CHF 83m forward swap (about the 10% who always extend).

CASE B · A GUARANTEE

A better fit is not a better replicating portfolio

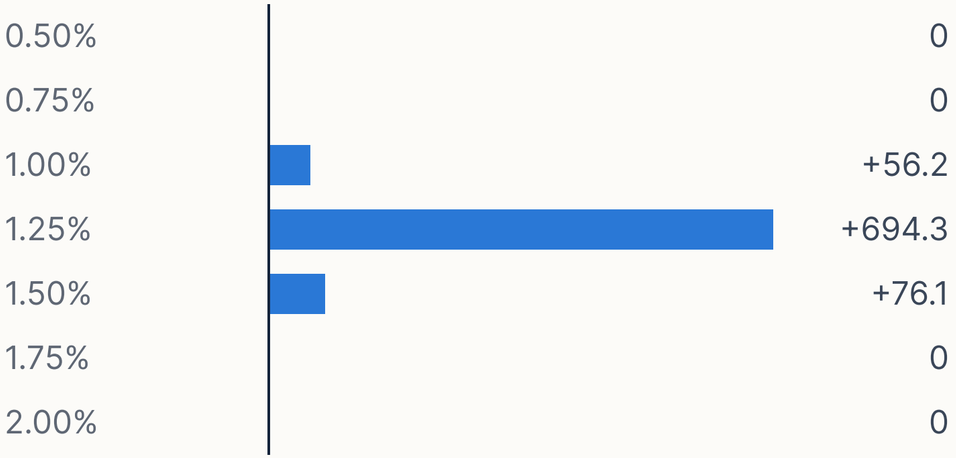
Unconstrained · seven strikes

R^2 0.99998 · value CHF 62.8m



Long-only options · same seven

R^2 0.9999 · value CHF 62.8m



Receiver notionals by strike, CHF m, one scale. Prefer few liquid instruments; test stability across refits, not only the in-sample R^2.

What insurers use replicating portfolios for

1. **Risk capital:** revalue the liabilities fast in thousands of one-year scenarios (SST: 99% expected shortfall; Solvency II: 99.5% value at risk)
2. **Hedging and ALM:** the replicating portfolio is the liability benchmark; the hedge buys part of it
3. **Communication:** liabilities in the language of the trading desk
4. **Steering:** limits, hedge effectiveness, P&L attribution

The nested-simulation problem

Without a proxy

10,000 one-year scenarios	×	1,000 valuation scenarios each	=	**10 million projections**

With a replicating portfolio

10,000 one-year scenarios	×	closed-form prices of a few dozen instruments	=	**no inner simulation**

Orders of magnitude, for illustration.

In practice: data, frequency, organisation, limits

Data

Scenario generator calibrated to the valuation-date curve and option vols; liability model points; behaviour from experience studies

Frequency

Full refit once or twice a year; quarterly roll-forward and fit checks; refit after big market moves

Organisation

Actuarial projections → risk (fit, capital) → investments (hedges)

independent validation; model governance

Limits and remedies

Path dependence	→ LSMC or richer instruments
behaviour	→ stresses in the fit
unstable weights	→ constraints

Many insurers use least-squares Monte Carlo (LSMC) instead of, or alongside, replicating portfolios (Pelsser and Schweizer, 2016).

BREAK · 10 MINUTES

The replicating portfolio of a guarantee is full of swaptions.

After the break: what a swaption is, how it is priced, who trades it, and why it hedges what swaps cannot.

PART 2 · 17:10–17:50

2

Swaptions

Mechanics, pricing, models and calibration, the market, hedging practice

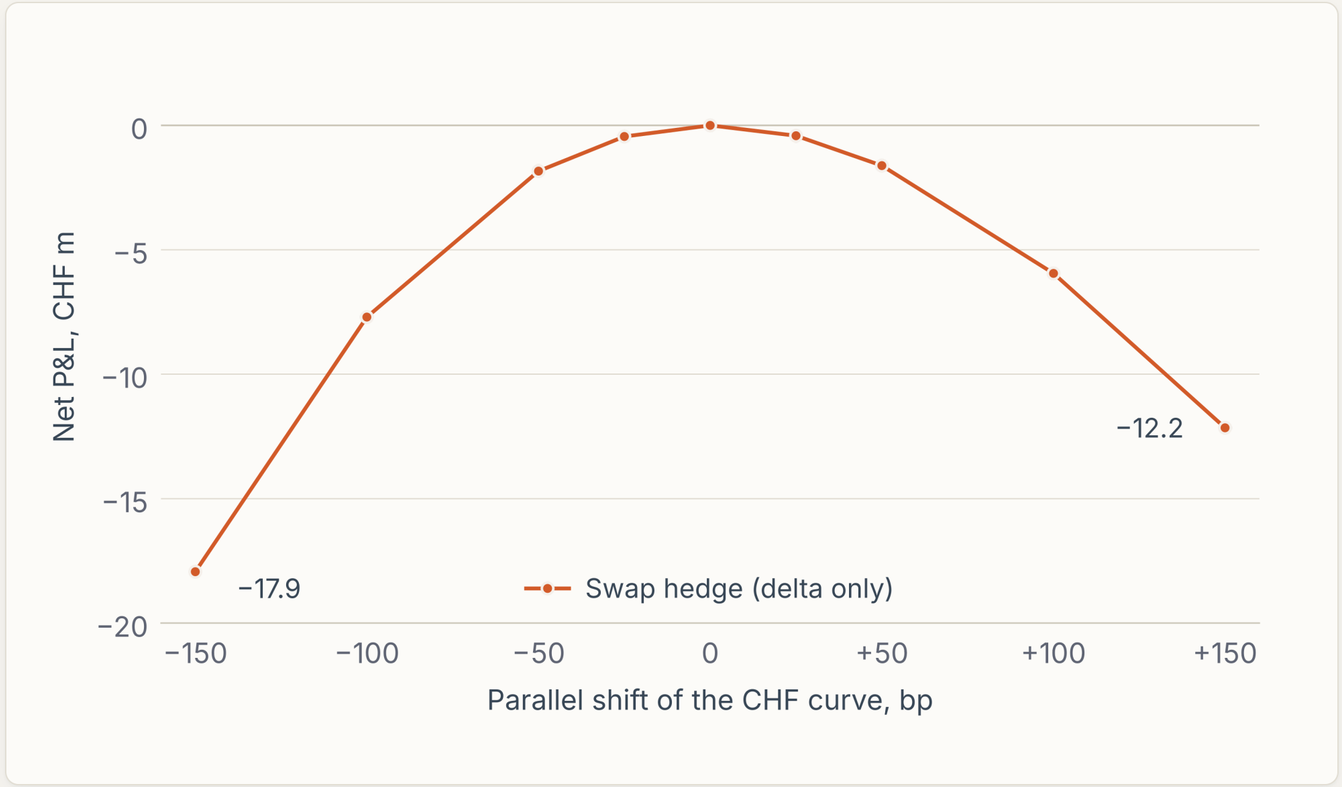
Why swaps are not enough

Case B, delta-hedged with a CHF 647m forward receiver swap (10y × 10y at 1.01%).

Swaps neutralise first-order sensitivity: DV01 and key rates.

The guarantee is a short option: negative gamma, negative vega.

Rates −150bp	**−CHF 17.9m**
Rates +150bp	**−CHF 12.2m**
Volatility +25%	**−CHF 13.5m**

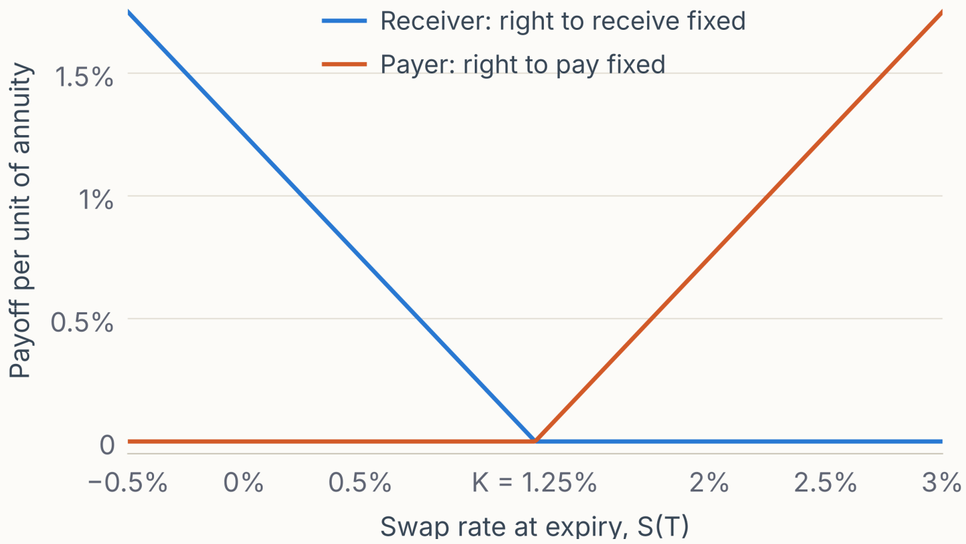


A swaption is an option on a forward-starting swap

$$\text{Payer}(T) = N A(T) \max(S(T) - K,\ 0) \qquad \text{Receiver}(T) = N A(T) \max(K - S(T),\ 0)$$

A(T): annuity of the underlying swap, the sum of its discounted fixed-leg periods. S(T): the swap rate at expiry.

- The right, not the obligation, to enter a swap at fixed rate K on expiry T
- **Payer:** right to pay fixed; gains when rates rise. **Receiver:** right to receive fixed; gains when rates fall
- Quoted expiry × tenor: 10y × 10y expires in 10 years into a 10-year swap
- Settled physically (enter the swap) or in cash
- In insurance: receivers protect guarantees, payers protect against mass surrender



Pricing: one change of numeraire away from Black

Annuity and forward swap rate	$A(t) = \sum_{i=1}^{n} \tau_i \, P(t, t_i) \qquad S(t) = \frac{P(t, T_0) - P(t, T_n)}{A(t)}$
Annuity as numeraire: S is a martingale	$\text{Payer}(0) = N A(0) \; \mathbb{E}^A[\max(S(T) - K, \, 0)]$
Black (lognormal S)	$\text{Payer} = N A(0)[F \, \Phi(d_1) - K \, \Phi(d_2)], \quad d_{1,2} = \frac{\ln(F/K) \pm \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$
Bachelier (normal S)	$\text{Payer} = N A(0)\left[(F - K) \, \Phi(d) + \sigma_N \sqrt{T} \, \varphi(d)\right], \quad d = \frac{F - K}{\sigma_N \sqrt{T}}$
Put–call parity	$\text{Payer} - \text{Receiver} = N A(0) \, (F - K)$

Since negative rates (CHF 2015–2022), swaption vols are mostly quoted as normal vols in bp a year, or as shifted lognormal: plain Black needs F and K above zero.

Worked example: the swaption inside Case B

Inputs

Receiver swaption	**10y × 10y**
Strike K	**1.25%**
Forward swap rate F	**1.012%**
Annuity A(0)	**8.762**
Normal volatility	**60bp a year**
Notional N	**CHF 1,000m**

$$\text{Receiver} = N A(0)\left[(K-F)\,\Phi(d) + \sigma_N\sqrt{T}\,\varphi(d)\right], \qquad d = \frac{K-F}{\sigma_N\sqrt{T}}$$

1. σ√T = 60bp × √10 = 190bp, so d = (1.25% − 1.012%) / 190bp = 0.125
2. **Receiver = CHF 77.26m** (773bp of notional): intrinsic 20.84m + time value 56.42m
3. Payer at the same strike: CHF 56.42m, by put–call parity
4. The same price in Black needs a 62% lognormal volatility
5. Delta −CHF 482k per +1bp in F; vega +CHF 1.10m per +1bp of normal volatility



intrinsic 20.84m time value 56.42m

The Hull–White model prices the same option at CHF 77.7m. Case B’s replicating portfolio costs 62.8m because not everyone exercises.

The volatility cube: expiry × tenor × strike

- Each point of the at-the-money matrix has its own smile across strikes, usually fitted with SABR (Hagan et al., 2002)
- Liquidity concentrates in shorter expiries and in EUR and USD
- Insurers need the long corner (10–20-year expiries, long tenors) and strikes near their guarantees: the thinnest data, sparse in CHF
- **Hence proxies, extrapolation and smoothing: model risk**

Expiry \ tenor	2y	5y	10y
1 year	69	68	65
5 years	67	65	63
10 years	65	63	60
15 years	62	60	58
20 years	59	58	55

At-the-money normal volatilities, bp a year, implied by our calibrated Hull–White model (illustrative). One factor makes the matrix smooth; market matrices have humps it cannot reproduce.

Interest rate models for insurers' scenario generators

$$dr_t = (\theta(t) - a\, r_t)dt + \sigma\, dW_t$$

Hull–White: the standard one-factor model that fits today's curve exactly

Why a model

Path-dependent liabilities need scenarios, and prices from those scenarios must match market prices: market-consistent and arbitrage-free.

Hull–White

θ(t) fits today's curve; mean reversion a and volatility σ are fitted to swaptions; bonds and swaptions price in closed form (Jamshidian, 1989).

Limits, and beyond

One factor: rates move together. Gaussian: negative rates possible, a feature in CHF. No smile. Richer: G2++, market models with stochastic volatility.

Calibrating the model to the market

Swaption	Market, bp	Model, bp	Error, bp
5y × 10y	64.0	62.7	−1.3
10y × 5y	62.0	62.9	+0.9
10y × 10y	60.0	60.3	+0.3
10y × 20y	57.0	55.9	−1.1
15y × 10y	57.0	57.9	+0.9
20y × 10y	55.0	55.4	+0.4

At-the-money normal volatilities. Fitted a = 0.0174 and σ = 70.7bp a year; root-mean-square error 0.9bp. Market volatilities are illustrative.

- Pick the calibration set to match the liability: expiries, tenors and strikes near the guarantee
- Weight by liquidity or vega; watch the parameters' stability from one date to the next
- Test: martingale and leakage tests, fit report, sensitivity to the calibration set
- Recalibrate at every valuation date; validate the scenario generator independently

Where the demand comes from

Who buys volatility

- **Life insurers and pension providers:** long-dated receivers against guarantees; structural and large
- **Insurers with surrender risk:** payers against a sharp rise in rates
- **Mortgage investors** with prepayment exposure, large in the US: receivers and other convexity hedges

Dealers: warehouse and recycle what is left; relative-value investors take the other side

Who supplies it

- **Issuers of callable bonds and structured notes:** the issuer owns the call and sells it on through dealers

⚠ Long-dated volatility carries a premium, and hedging demand peaks when rates fall: the hedge is dearest when it is needed most.

Using swaptions in practice

Designing the hedge

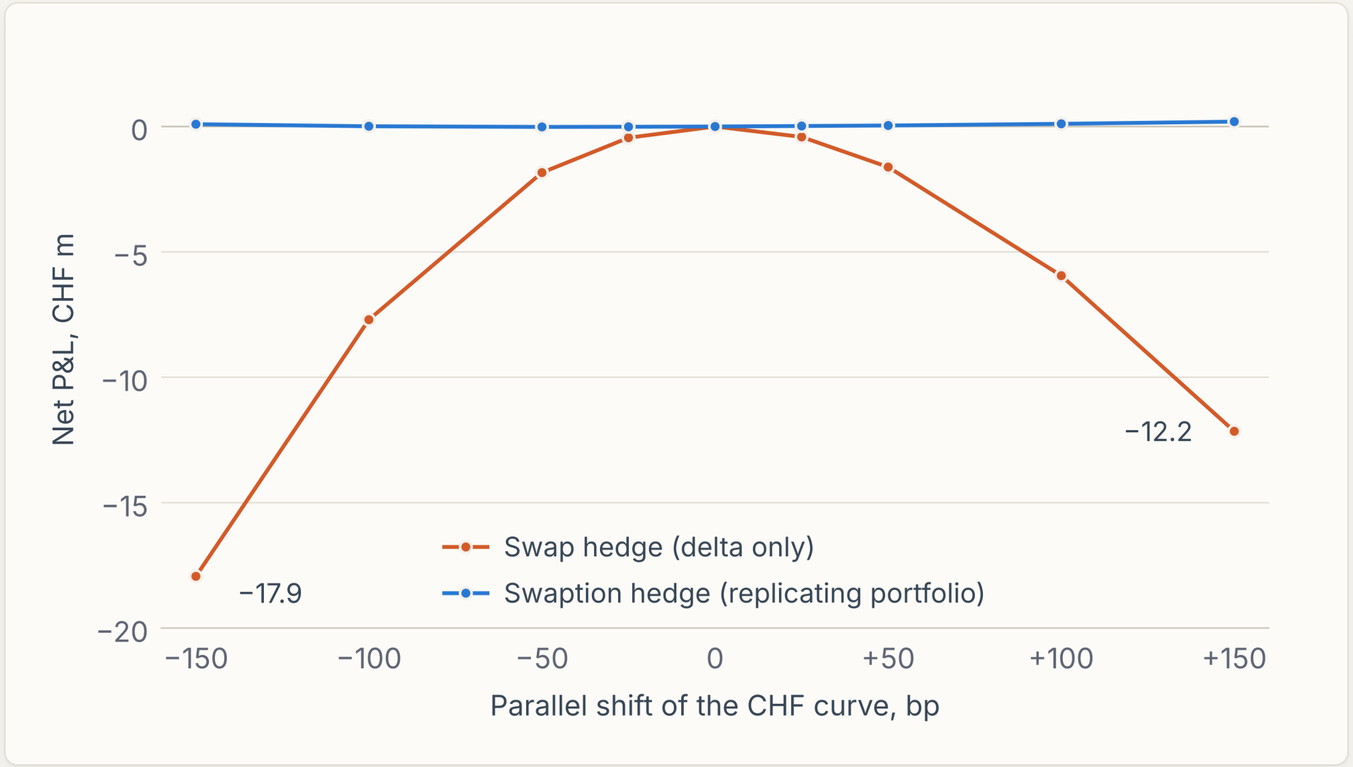
1. **Map** to tradable swaptions: expiry when the option bites, tenor for the guaranteed cash flows after it, strike around the guarantee
2. **Size** by notional, or by matching vega and gamma
3. **Ladder** expiries; roll as time passes

Running it

- **Premium** up front (Case B: about CHF 63m): budgets, partial hedge ratios
- **Collars** (buy receivers, sell payers) cut the cost but add rising-rate risk: dangerous if surrenders rise too
- **Monitor** DV01, key rates, gamma and vega; rebalance on triggers

OTC swaptions are collateralised under ISDA and CSA agreements: margin calls create liquidity needs. That is a topic in its own right: counterparty risk and collateral.

Case B: the hedge that works



Net, CHF m	Swaps	Swaptions
−150bp	−17.9	+0.1
−50bp	−1.8	0.0
+50bp	−1.6	0.0
+150bp	−12.2	+0.2
Vol +25%	−13.5	+0.2

Inside the model the replicating portfolio hedges almost perfectly. In reality the residual is model, behaviour and basis risk, not zero.

WRAP-UP

Limits and pitfalls

Model risk

Scenario generator, calibration set, extrapolation beyond liquid data

Behaviour risk

Surrenders and extensions are not rational exercise, and they shift in stress

Basis risk

Regulatory discount curves (extrapolated in Solvency II, set by FINMA in the SST) versus swaps

Liquidity

Collateral calls on hedges: UK pension funds' LDI strategies in September 2022

Accounting

Hedges under IFRS 9, liabilities under IFRS 17; regulatory and economic views can disagree

Regime change

Negative rates forced new models and quoting conventions; the next regime will too

WRAP-UP

Four things to take home

1. Life insurers typically lose when rates fall, and their liabilities contain options sold to policyholders.

2. Replicating portfolios turn insurance liabilities into tradable instruments: zero-coupon bonds for fixed cash flows, bonds plus swaptions for guarantees. One proxy serves valuation, capital and hedging.

3. A swaption is an option on a forward swap rate; under the annuity measure it is priced with Bachelier or Black.

4. Swaps hedge delta; only options hedge gamma and vega, which makes insurers structural buyers of long-dated receivers. Every hedge keeps model, behaviour and basis risk.

WRAP-UP

Further reading

Replicating portfolios

Cambou, M. & Filipović, D. (2018). Replicating portfolio approach to capital calculation. *Finance and Stochastics* 22(1), 181–203.

Pelsser, A. & Schweizer, J. (2016). The difference between LSMC and replicating portfolio in insurance liability modeling. *European Actuarial Journal* 6(2), 441–494.

Natolski, J. & Werner, R. (2014). Mathematical analysis of different approaches for replicating portfolios. *European Actuarial Journal* 4(2), 411–435.

Wüthrich, M. V. (2016). *Market-Consistent Actuarial Valuation*, 3rd ed. Springer.

Guarantees as options

Boyle, P. & Hardy, M. (2003). Guaranteed annuity options. *ASTIN Bulletin* 33(2), 125–152.

Pelsser, A. (2003). Pricing and hedging guaranteed annuity options via static option replication. *Insurance: Mathematics and Economics* 33(2), 283–296.

Swaptions and rate models

Brigo, D. & Mercurio, F. (2006). *Interest Rate Models – Theory and Practice*, 2nd ed. Springer.

Hull, J. C. (2022). *Options, Futures, and Other Derivatives*, 11th ed. Pearson.

Hagan, P. S., Kumar, D., Lesniewski, A. S. & Woodward, D. E. (2002). Managing smile risk. *Wilmott Magazine*, September, 84–108.

Hull, J. & White, A. (1990). Pricing interest-rate-derivative securities. *Review of Financial Studies* 3(4), 573–592.

Jamshidian, F. (1989). An exact bond option formula. *Journal of Finance* 44(1), 205–209.

Thank you

Questions?

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Slides: ilkeraslan.github.io

APPENDIX

Case data and conventions

Tenor	Par swap	Zero, cont.
1 year	0.16%	0.160%
2 years	0.38%	0.380%
5 years	0.60%	0.600%
10 years	0.76%	0.762%
15 years	0.86%	0.864%
20 years	0.88%	0.884%
30 years	0.82%	0.816%

Curve: CHF SARON swaps, annual fixed against compounded SARON; interdealer mids at the close of 19 October 2026 (BlueGamma). Single-curve discounting; missing tenors interpolated linearly in par rates.

Case A: CHF 500m paid at the end of years 1–10 in shares of 22, 18, 14, 11, 9, 7, 6, 5, 4 and 4%; key rates at 2, 5 and 10 years with triangular shifts of zero rates.

Case B: Hull–White (G1++ form) fitted to the curve and to six swaption volatilities; values under the T-forward measure; Jamshidian prices; 5,000 scenarios in and 5,000 out of sample. Share extending: 10% + 80% × logistic((1.25% − S)/25bp).

Illustrative: swaption volatilities, the reserve pattern and the policy data. Every figure comes from one reproducible script.